

ТРИ КЛАССА ТРЕУГОЛЬНИКОВ

1 Определения и вводные задачи

2 Main problems

We will call the circle touching two sides of a triangle and its circumcircle *the semiincircle*. The semiincircle of triangle ABC touching sides AB and AC will be denoted as ω_a .

13A. a) Which is the touching point of ω_a and Ω ?

b) Prove that quadrilateral $H'_a C' AB'$ is harmonic.

Consider the circle with center B touching CH and the circle with center C touching BH . Denote them as Γ_b and Γ_c respectively.

14A. Prove that Γ_b and Γ_c touch at some point lying on BC .

Denote the touching point from the previous problem as X .

15A. Prove that I and X lie on $H'_a A''$.

16A. Prove that lines IH , BC and $H'_a A'$ concur.

Denote the concurrency point from the previous problem as Y .

17A. Prove that Y is the center of positive homothety mapping Γ_b to Γ_c .

18A. Prove that the second tangents to Γ drawn from X and Y touch Γ at its intersection points with the line AH .

19A. Prove that HA_0 and $A'X$ meet at Ω .

20A. Prove that HA_0 , $H'_a O$ and AA' concur.

21A. a) Prove that the concurrency point from the previous problem is the center of ω_a .

b) Let R and S be the common points of OI with ω_a , and T be the center of ω_a . Find angle STR .

6B. IH bisects AC .

7B. B' lies on $A_b C_b$.

8B. OI passes through B_b .

9B. A_a, B_b, C_c, H_b are concyclic.

Definition. Line l is called *the polar* of point P wrt the circle with center O and radius R , if ray OP meets l , is perpendicular to it, and the distance from O to l is equal to $\frac{R^2}{OP}$. A triangle is called *autopolar* wrt the circle, if its sidelines are the polars of opposite vertices.

- 9C. The triangle $A_cB_cC_c$ is autopolar wrt Ω .
- 10C. The intersection point of the lines that are tangent to the circle Ω at the points A and B lies on the line A_cB_c .
Let AL_a, BL_b be the bisectors of angles A and B of triangle ABC .
- 11C. O lies on the line L_aL_b .
- 12C. Let the line L_aL_b intersects the circle Ω at the points X and Y .
a) The triangle I_cXY is equilateral.
6) Determine, which point is the center of this triangle.
- 13C. I lies on the line H_aH_b .
- 14C. The line OI contains H_c .
- 15C. (A.Golmakani, Iran) Let the lines AL_a and BL_b intersect the line, tangent to the circle Ω at point C , at points P and Q respectively. Then the line OI passes through the midpoint of the segment PQ .
It is well known, that in any triangle symmedians (the lines, symmetric to the medians of this triangle with respect to its angle bisectors) have the common point. This point is called *Lemoann point* L .
- 16C. The lines OG_c and LI_c are parallel.
- 17C. The polar of the point G_c with respect to the excircle passes through L .

3 Extra problems

- Each of the circles ω_1 and ω_2 contains the center of the other circle and they intersect each other at points P and Q . The lines, passing through C tangent to ω_1 intersect ω_1 at the second time at the points A and B . Prove that $AB \parallel PQ$. In a convex quadrilateral $ABDC$ $\angle ABD = \angle ACD = 90^\circ$ and $DA = DB + DC$. Prove that AD divides the triangle ABC on two polygons with equal perimeters.
- Two circles of radii 1 intersect each other at points X Y . $XY = 1$. CA and CB are tangent lines to one of these circles from point C lying on the other one of these circles. Line CB meets the first circle for the second time at point A' . Determine the length AA' .

Due to *Poncelet theorem* there exist infinitely many triangles with fixed circumcircle and excircle. Consider all such triangles ABC .

- 18C. a) The sidelines of all correspondent triangles $A_cB_cC_c$ touch some fixed hyperbola.
- b) Which is the touching point of this hyperbola with line A_cB_c ?
- c) Determine the foci of this hyperbola.
- 19C. a) Prove that the common points of lines AB and A_cB_c , AC and A_cC_c , BC and B_cC_c lie on some line touching the hyperbola.
- b) Determine the point of tangency.
- BC. Line OI meets one of sides of a triangle at the base of the correspondent altitude, and the second one at its touching point with the respective excircle. Find the angle between these sides.